PG – 364

II Semester M.Sc. (CBCS) Examination, June 2015
MATHEMATICS
M 203 T : Topology – II

Time : 3 Hours   Max. Marks : 70

Instructions : *i) Answer **any five** questions.*
*ii) **All** question carry **equal** marks.*

1. a) Define a compact space with an example. If E is a subset of a subspace (Y, T *) of a topological space (X, T), then prove that E is T * – compact if and only if it is T -compact.

   b) Every countable open cover of (X, T) has a finite subcover. Then prove that (X, T) is countably compart.

   c) Prove that every closed subset of a locally compact space is locally compact. **(6+4+4)**

2. a) Define :

   i) A First Axiom Space (FAS).

   ii) Second Axiom Space (SAS)

   Prove that every SAS is a separable space. Is the converse true ? Justify.

   b) Show that every second countable space is Lindeloff.

   c) Show that a compact metric space is totally bounded. Is the converse false ? Justify your answer. **(7+3+4)**

3. a) Show that a mapping f : z $\rightarrow$ X × Y is continuous iff $\Pi_x \circ f$ and $\Pi_y \circ f$ are continuous.

   b) Prove that

   i) X × Y is locally connected iff X and Y are locally connected.

   ii) X × Y is completely regulars iff X and Y are completely regular. **(5+5+4)**

4. a) Prove that an infinite set with the co-finite topology is a T_1-space.

   b) Define T_2-space show that T_1-space does not implies T_2-space.

   c) Show that a compact subset of a Hausdorff space is closed. **(4+5+5)**

5. a) Define a regular space. Prove that a regular space need not be a T_1-space.

   b) Prove that T_3-space is both hereditary and topological property. **(6+8)**

6. a) Define a normal space. Prove that a space (X, T) is normal iff given any open set G and a closed set $F \subseteq G$, there exists an open set G* such that $F \subseteq G^* \subseteq \overline{G^*} \subseteq G$.

   b) Prove that a compact Housdorff space is normal. **(7+7)**

7. a) State and prove tietze's extension theorem.

   b) Define a completely normal space and prove that completely normality implies normality. **(10 + 4)**

8. a) Prove that a space is completely normal iff every subspace is normal.

   b) Prove that every metric space is completely normal. **(7+7)**